$*54{\cdot}43.\quad \vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54{\cdot}26 . \supset \vdash :. \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

[*51·231] $\equiv . \iota`x \cap \iota`y = \Lambda .$

[*13·12] $\equiv . \alpha \cap \beta = \Lambda \qquad (1)$

$\vdash . (1) . *11{\cdot}11{\cdot}35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \qquad (2)$

$\vdash . (2) . *11{\cdot}54 . *52{\cdot}1 . \supset \vdash . \text{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

Bertrand Russell, Alfred Whitehead, and Mathematical Logic

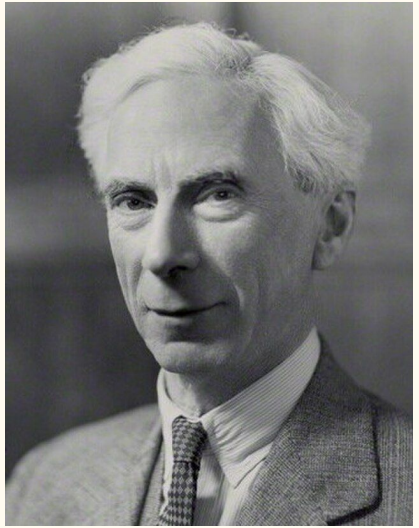
Stephanie Spoto

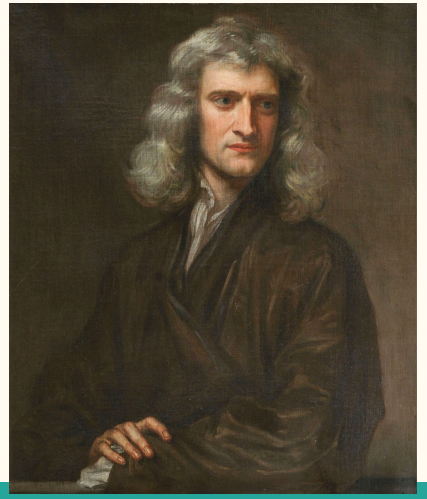
Principia Mathematica

- A work of mathematical logic in three-volumes (1910-1913)
- Written by Alfred North Whitehead (1861-1947) and Bertrand Russell (1872-1970)
- Both were working separately on the project of logicism: the idea that all mathematics can be reduced down into system of symbolic logic
- Decided to combine their powers to collaborate on *Principia Mathematica*
- Mathematical concepts come from pure logic/logical premises

Historical Foundations

- Name their book after Isaac Newton's *Philosophiæ Naturalis Principia Mathematica* (1687) → want their work to do for logic what Newton's book did for physics
- Early 1900s: Crisis in mathematics which was caused by various paradoxes that needed resolving, such as **Russell's Paradox**: "Does the set of all sets that are not members of themselves contain itself?"
- Split in analytic and continental philosophy → with analytic philosophy turning away from the metaphysics, existentialism, etc. that is taken up by continental philosophers

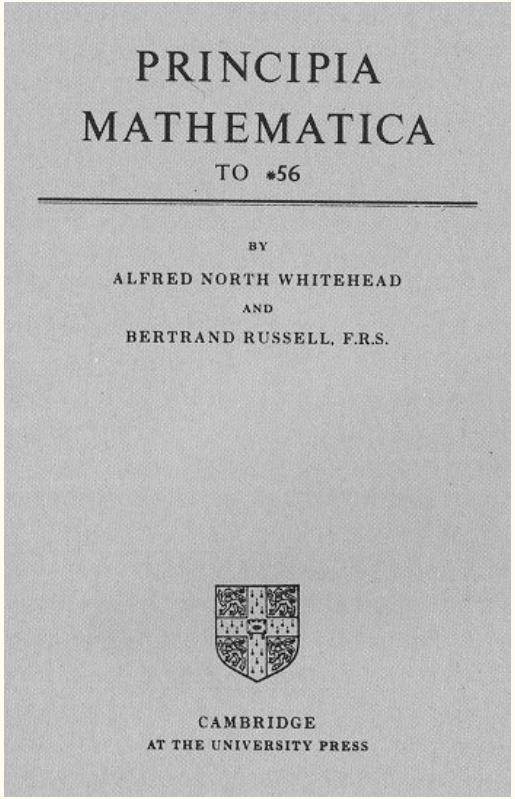




Overgrows original smaller scope

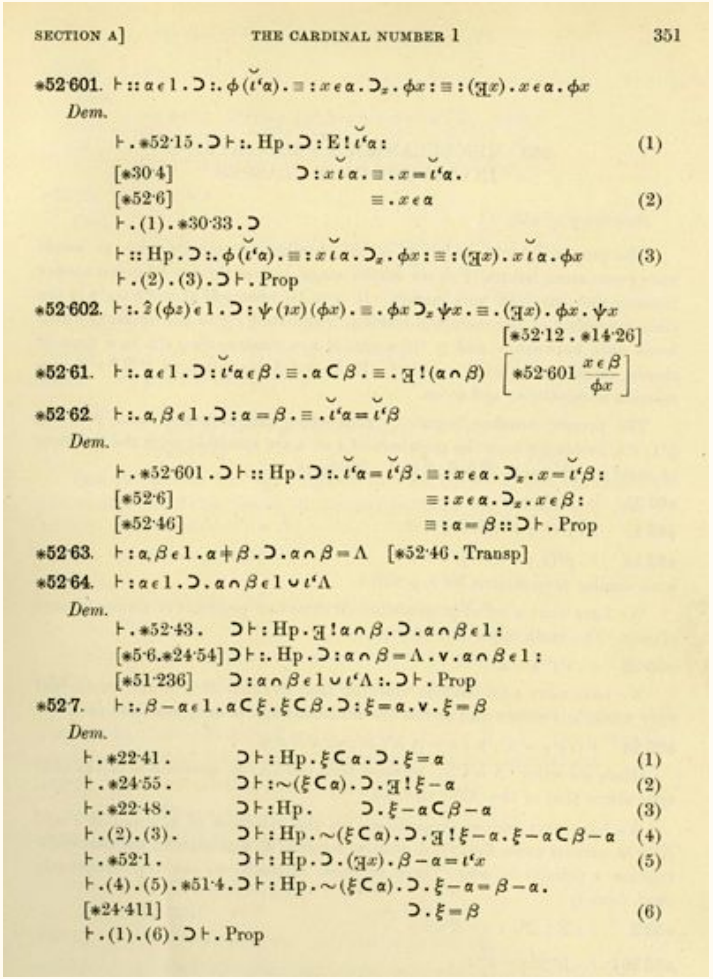
- Originally conceived as a smaller project, a second volume of a book that Russell had been working on, the *Principles of Mathematics* (1903)

"The present work was originally intended by us to be comprised in a second volume of Principles of Mathematics ... But as we advanced, it became increasingly evident that the subject is a very much larger one than we had supposed; moreover on many fundamental questions which had been left obscure and doubtful in the former work, we have now arrived at what we believe to be satisfactory solutions."



Aim of work

1. to analyse to the greatest possible extent the ideas and methods of mathematical logic and to minimise the number of primitive notions, axioms, and inference rules;
2. to precisely express mathematical propositions in symbolic logic using the most convenient notation that precise expression allows;
3. to solve the paradoxes that plagued logic and set theory at the turn of the 20th century, like Russell's paradox



SECTION A] THE CARDINAL NUMBER 1 351

$*52{\cdot}601.\ \vdash::\alpha\in 1.\supset:.\phi(\breve{\iota}`\alpha).\equiv:x\in\alpha.\supset_x.\phi x:\equiv:(\exists x).x\in\alpha.\phi x$

Dem.

$\vdash.*52{\cdot}15.\supset\vdash:.\mathrm{Hp}.\supset:\mathrm{E}!\breve{\iota}`\alpha:$ (1)

[*30·4] $\supset:x\,\breve{\iota}\,\alpha.\equiv.x=\breve{\iota}`\alpha.$

[*52·6] $\equiv.x\in\alpha$ (2)

$\vdash.(1).*30{\cdot}33.\supset$

$\vdash::\mathrm{Hp}.\supset:.\phi(\breve{\iota}`\alpha).\equiv:x\,\breve{\iota}\,\alpha.\supset_x.\phi x:\equiv:(\exists x).x\,\breve{\iota}\,\alpha.\phi x$ (3)

$\vdash.(2).(3).\supset\vdash.\mathrm{Prop}$

$*52{\cdot}602.\ \vdash:.\hat{z}(\phi z)\in 1.\supset:\psi(\iota x)(\phi x).\equiv.\phi x\supset_x\psi x.\equiv.(\exists x).\phi x.\psi x$ [*52·12 . *14·26]

$*52{\cdot}61.\ \vdash:.\alpha\in 1.\supset:\breve{\iota}`\alpha\in\beta.\equiv.\alpha\subset\beta.\equiv.\exists!(\alpha\cap\beta)$ $\left[*52{\cdot}601\,\frac{x\in\beta}{\phi x}\right]$

$*52{\cdot}62.\ \vdash:.\alpha,\beta\in 1.\supset:\alpha=\beta.\equiv.\breve{\iota}`\alpha=\breve{\iota}`\beta$

Dem.

$\vdash.*52{\cdot}601.\supset\vdash::\mathrm{Hp}.\supset:.\breve{\iota}`\alpha=\breve{\iota}`\beta.\equiv:x\in\alpha.\supset_x.x=\breve{\iota}`\beta:$

[*52·6] $\equiv:x\in\alpha.\supset_x.x\in\beta:$

[*52·46] $\equiv:\alpha=\beta::\supset\vdash.\mathrm{Prop}$

$*52{\cdot}63.\ \vdash:\alpha,\beta\in 1.\alpha\neq\beta.\supset.\alpha\cap\beta=\Lambda$ [*52·46 . Transp]

$*52{\cdot}64.\ \vdash:\alpha\in 1.\supset.\alpha\cap\beta\in 1\cup\iota`\Lambda$

Dem.

$\vdash.*52{\cdot}43.\ \supset\vdash:\mathrm{Hp}.\exists!\alpha\cap\beta.\supset.\alpha\cap\beta\in 1:$

[*5·6.*24·54] $\supset\vdash:.\mathrm{Hp}.\supset:\alpha\cap\beta=\Lambda.\vee.\alpha\cap\beta\in 1:$

[*51·236] $\supset:\alpha\cap\beta\in 1\cup\iota`\Lambda:.\supset\vdash.\mathrm{Prop}$

$*52{\cdot}7.\ \vdash:.\beta-\alpha\in 1.\alpha\subset\xi.\xi\subset\beta.\supset:\xi=\alpha.\vee.\xi=\beta$

Dem.

$\vdash.*22{\cdot}41.\ \supset\vdash:\mathrm{Hp}.\xi\subset\alpha.\supset.\xi=\alpha$ (1)

$\vdash.*24{\cdot}55.\ \supset\vdash:\sim(\xi\subset\alpha).\supset.\exists!\xi-\alpha$ (2)

$\vdash.*22{\cdot}48.\ \supset\vdash:\mathrm{Hp}.\ \supset.\xi-\alpha\subset\beta-\alpha$ (3)

$\vdash.(2).(3).\ \supset\vdash:\mathrm{Hp}.\sim(\xi\subset\alpha).\supset.\exists!\xi-\alpha.\xi-\alpha\subset\beta-\alpha$ (4)

$\vdash.*52{\cdot}1.\ \supset\vdash:\mathrm{Hp}.\supset.(\exists x).\beta-\alpha=\iota`x$ (5)

$\vdash.(4).(5).*51{\cdot}4.\supset\vdash:\mathrm{Hp}.\sim(\xi\subset\alpha).\supset.\xi-\alpha=\beta-\alpha.$

[*24·411] $\supset.\xi=\beta$ (6)

$\vdash.(1).(6).\supset\vdash.\mathrm{Prop}$

The Primacy and New Role of Philosophy

- Wants to show that mathematics actually has philosophy as its foundation
- Logic (originates in philosophy) is the source of mathematics
- Mathematics truths are logical truths
- Mathematical objects and numbers can be defined using logic alone
- Project called **logicism**
- Logicism became one of the cornerstones of **analytic philosophy**
    - Avoid broader questions like “what is reality?” and instead focuses on questions that can be understood in smaller or more well defined terms
    - Instead of “What is truth?” might ask “What conditions make a statement true?”

The Analytic-Continental Divide in Western Philosophy – Sort Of. . .

ANALYTIC PHILOSOPHY

- Develops and is found predominantly in Anglophone countries and Scandinavia
- Begins from reaction against British idealism, with focus on logic, language, empiricism, the sciences, etc.
- Ideals of clarity, objectivity, rationality
- Provides analysis of concepts, arguments, intuitions, uses of language, etc.
- More aligned with the natural sciences, mathematics, some social sciences
- Criticized as ahistorical, myopic, superficial, imperialistic, unimaginative, and so on.
- Canonical Figures: Frege, Russell, Carnap, Wittgenstein, Ayer, Ryle, Quine, Davidson, Lewis (or is this rather poor list still "canonical"?)

CONTINENTAL PHILOSOPHY

- Develops and is found in Germany, France, Italy, Spain, South America, etc.
- Stems from multiple sources in post-Kantian European philosophy, focuses on a vast variety of often interconnected matters, topics, issues.
- Ideals of critique, subjectivity, imagination
- Provides descriptions of experience, textual analysis, critique or liberation, etc.
- More aligned with the humanities, the fine arts, some social sciences.
- Criticized as incomprehensible, relativist, trendy or faddish, obscure, polemical, poetic, and so on
- Canonical Figures: Hegel, Marx, Nietzsche, Husserl, Heidegger – or maybe Foucault, Deleuze, Agamben, Butler? – or . . .

And

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

Or

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

If … then

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

Not

p	$\sim p$
T	F
F	T

Demonstrate the validity of this argument. Reduce the word problem to symbolic logic and then offer a proof, citing the rules of algebra or logical inference, that allow you to conclude that the conclusion must be true.

- She is a Math Major or a Computer Science Major.
- If she does not know discrete math, she is not a Math Major.
- If she knows discrete math, she is smart.
- She is not a Computer Science Major.
- Therefore, she is smart.

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Let p: math major, q: computer science major, r: discrete math, s: smart

The arguments are: $p \vee q\,;\ \neg r \longrightarrow \neg p\,;\ r \longrightarrow s\,;\ \neg q\,;\ \therefore s$

Problems with publication

- Published by Cambridge University Press
- Projected a sales deficit of £600 → publishers offered to pay for some of the deficit, Royal Society paid another £200, this left £100 for the authors to pay
- There was a planned fourth volume, but the authors were burnt out and gave up the project

“I can remember Bertrand Russell telling me of a horrible dream. He was in the top floor of the University Library, about A.D. 2100. A library assistant was going round the shelves carrying an enormous bucket, taking down books, glancing at them, restoring them to the shelves or dumping them into the bucket. At last he came to three large volumes which Russell could recognize as the last surviving copy of Principia Mathematica. He took down one of the volumes, turned over a few pages, seemed puzzled for a moment by the curious symbolism, closed the volume, balanced it in his hand and hesitated....”

G. H. Hardy, *A Mathematician's Apology* (1940)

Legacy

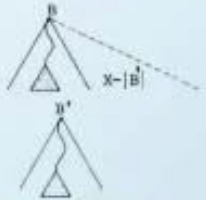
- Became a foundational and mandatory subject for all philosophy majors at American universities :(
- Formalized the split between the continental and analytic traditions
- Type Theory (developed to avoid paradoxes) becomes base approach for modern computer science → specifically in programming languages
- Failed: The attempt to totally reduce mathematics down to logic is a project
- But Neologicians use it to attempt to show how much of mathematics can be reduced to logic

STOP DOING LOGIC

- ARGUMENTS WERE NOT SUPPOSED TO BE FORMALIZED
- SO MANY RULES yet NO REAL-WORLD USE FOUND for going beyond MODUS PONENS
- Wanted to prove things anyway for a laugh? We had a tool for that: It was called "INDUCTION"
- "Hello, how are $\Diamond a_{n+1}/\neg(a_{n+1} \rhd \neg c_{n+1})$ doing? Isn't the weather $\forall \phi\,(\Box_{\Gamma}\varphi \rightarrow \Box_{U\cup\{\mathrm{Con}^r(V)\}}\varphi^J)$ today?" - Statements dreamed up by the utterly Deranged

LOOK at what Logicians have been demanding your Respect for all this time, with all the arguments and languages we built for them
(This is REAL Logic, done by REAL Logicians):

?????



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"""""$You\ h\forall v\exists\ to\ b\exists\ logic\forall l$"""""

They have played us for absolute fools